Model Upgrading T0 Augment Linear Model Capabilities Into Nonlinear Regions

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ABSTRACT

Identification of nonlinear dynamical systems have enjoyed significant progression over the past few years with the outcome of various developed identification methods, however there is still no generalised method applicable to structures with arbitrary nonlinearity. In the analysis of nonlinear dynamical system, it is essential to establish accurate and reliable tools that are capable of estimating the parameters from measured data for both the linear and nonlinear system. This paper presents a modular framework approach for upgrading a valid linear finite element structural model to accommodate any nonlinearities present in a system. To validate the efficiency of the proposed method, numerical and experimental studies are conducted on a "Multiple Beam Test Structure", the method uses an iterative process to upgrade the nonlinear terms in the system. The results are verified by comparing predicted new response with measured data.

KEYWORDS: Nonlinearities, Model Upgrading, Finite Element, Structural Models and Framework

1. Introduction

Nonlinear system identification has received a lot of attention over the last few years up till date with examples such as continuous structures with localised geometrical nonlinearity [1, 2], a compressive review on the types of nonlinearity and methods of nonlinear system identification can be found in [3]. Identification of nonlinear multi-degree-of-freedom (MDOF) lumped parameters was also presented in [4, 5]. The identification of weak nonlinearities was also studied on more complex structures, example of this can be found [6] where a strategy for non-linear modal identification of weak nonlinear effects on large aircraft structures was presented. An aluminium plate attached with two stores used to illustrate the behaviour of a wing and an engine suspended by a means of nonlinear pylon also displayed a presence of weak nonlinearities during a vibration test, the results obtained illustrated some hardening characteristics as show in [7]. Similar study was also carried out on a large helicopter with the identification of a weak nonlinear softening behaviour on one of the vibration modes as shown in [8].The current process of modelling nonlinearity in engineering structures is by including some corresponding nonlinear elements in the mathematical models which describes the nonlinear system. For this type of case study, the parameters of these nonlinear elements are usually specific and can only be obtained from an experimental test or through an updating process.

However, engineers today are frequently being challenged and confronted with the presence of nonlinear behaviour in large structures where they are not easy to locate. Examples of case studies where nonlinearity have been noticed in aerospace structures can be found in [9] where nonlinearity was detected and reported at the elastomeric mounts supporting the four turboprop engines of the aircraft during the Ground Vibration Test (GVT) of the Airbus A400M aircraft designed for military purpose. The F-16 fighter aircraft also showed some nonlinearity behaviour at wing-to-payload mounting interface of the aircraft when a similar GVT was conducted [10]. Nonlinearities were also detected on the Cassini spacecraft due to the presence of gaps in the support of the Huygens probe [11]. More case studies on the presences of nonlinearities in engineering structures can be found in the literature, this shows that there is a requirement in designing appropriate tools, or framework, which are capable of dealing with cases where nonlinearities are present in today's engineering structures.

In this paper authors seek to propose a modular framework for upgrading a linear system to capture any nonlinearities present in a structure, the framework is based on establishing an underlying linear model or properties of the structure through an optimisation process. The nonlinear identification is conducted by using a process which entails *detection, localisation, characterisation and quantification* of the nonlinearity in the system. The upgrading process regards the inclusion of missing physical terms which enable a finite element model to predict responses for any loading conditions.

Review on Nonlinear Identification Methods

In a comprehensive review presented by Kerschen.,et., al, in [3], seven methods have been identified as the most popular methods used in the analysis of nonlinear system. These methods are:

- Time domain methods
- Frequency domain methods,
- Modal methods,
- Linearization methods,
- Time-frequency analysis,
- Black-box modelling
- Structural modelling updating

A survey of these seven identification methods are discussed fully in[3], in addition the authors of this paper have undertaken a statistical analysis on the practical application of these methods most especially in the field of structural dynamics. In the analysis small structures are categorised as examples of simulation or experimental work which has been carried out on beams, plates, lap joints and small masses connected together with a nonlinear device using the associated technique. Large structures are categorised as automotive shock absorbers, full scales shear wall structures, helicopter blades, aircraft wings, ailerons, SmallSat Spacecraft, and a full-scaled small aircraft.

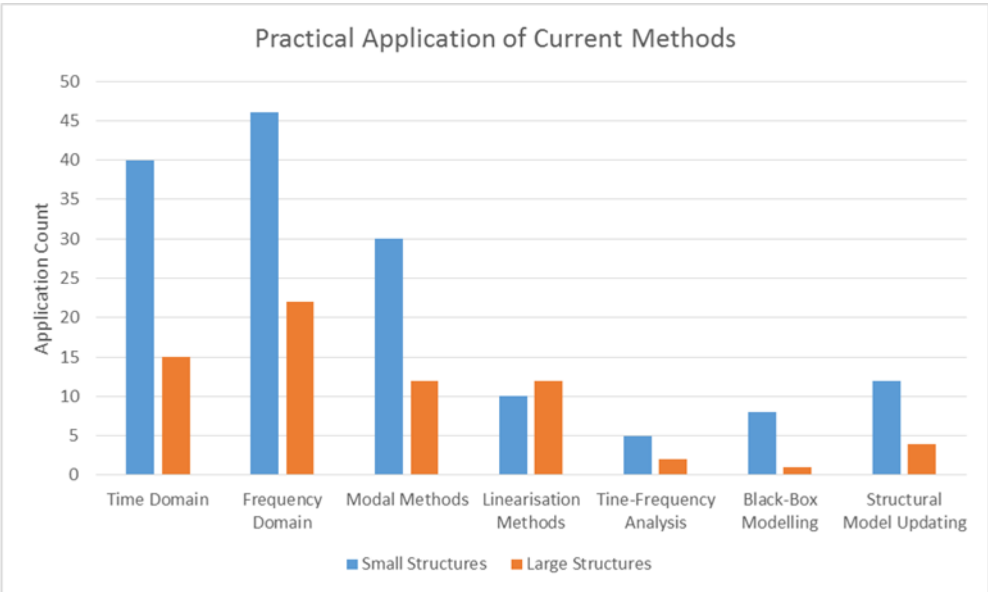

Figure 1-Practical Application of Current Identification Methods, subcategorised by small and large structures

Despite the level of development in nonlinear identification methods, the chart presented in Figure 1 indicates that only a small percentage of these methods are practically suitable for structures modelled with large degrees of freedom (DOF) and localised nonlinearity. For clarity, the above chart is intended to show a comparison of how each method has been applied in the identification of nonlinearity in different industrial application. An important criterion in the industrial environment is that a method is acknowledged as a good method if it can be applied to a real life structure or system. In most cases the direct application of these methods are not always straightforward due to the level of mathematical algorithms that these methods are founded upon.

In the field of structural dynamics, the basic concept behind nonlinear identification is that every structure is considered to have an underlying linear model (ULM) where the response of a structure obeys the homogeneity principle up to a certain level before shifting into nonlinear region where superposition is no longer valid. Regardless of the extensive literature on linear and nonlinear identification, there is little or no work on developing methodologies that are capable of enhancing validated linear structural models to accommodate discrete and localised nonlinearities. Hence this paper is aimed at presenting a practical application of a modular framework where an optimised linear FE structural model is upgraded to operate normally in the presence of discretized and localised nonlinearities.

2. Identification Process of Nonlinear System

Another way to classify the current literature on nonlinear identification is based on the processes involved in the identification of a nonlinear system. In this context, *Identification* process is defined as the number of steps or procedures required for the complete identification of a nonlinear system. Arguably, in structural dynamics, the identification process is often developed based on the type and source of nonlinearity associated with the structure or system. This aspect of nonlinear identification has also received the development of several procedures and strategies that is capable of providing a successful nonlinear identification. A more generic identification process, which seeks to provide answers to some typical questions concerning nonlinear structures, is to obtain answers to the following ones:

(i) Does the structure exhibit a nonlinear behaviour?
(ii) What is the type and source of nonlinearity?
(iii) What is the strength of the nonlinearity?

However an updated version of this process which incorporates modal testing into each step definition is presented in an opinion paper in [12]. This new updated process is tailored to answer industrial practical problems where structural integrity is of great concern. More often industries are being challenged with cases where their proposed structural linear FE model fails to produce reliable predicted response due to the presence of some nonlinear features in the structure. The underlying principle for this new updated process is that an improved modal testing and analysis identification process might be an appropriate way of solving some of the challenges that industries are currently facing in this area. The major definition of these processes are:

1. *Modal Testing+* **Detection**: to determine the strength of nonlinearity from measured response.
2. *Modal Testing+* **Characterisation**: to determine the type of nonlinearity.
3. *Modal Testing+* **Location**: to locate regions containing nonlinear features.
4. *Modal Testing+* **Quantification**: to quantify the nonlinear features.

In this context detection, characterisation, location and quantification are defined as:
Detection is used as a form of indicator based on the measured response to ascertain that there is some effect of nonlinearity which cannot be neglected and at that stage the structural model is no longer classified as a linear model.
Location is mainly to determine where the nonlinear features are located in the structural model and also which are the corresponding degrees of freedom (DOF) in the FE model.
Characterisation is referred to the physical origins of the nonlinear features in the structure and most importantly the source of the nonlinear effect i.e stiffness or damping of the structural model.
Quantification involves searching for the coefficient of the characterised nonlinear term.

3. Modular Framework and Upgrading Approach

In this era of structural modelling where finite element method is widely used, there is still a great concern with incorporating valid linear FE models with identified nonlinearities in a structure. This modular framework is structured mainly to enhance an optimised linear structural model to accommodate any form of nonlinearities. A large majority of the procedure in this framework is based on the physical modelling approach while some subsection like the damping ratio estimation is based on the traditional modal space approach. The physical modelling approach has the abilities to identify nonlinearities in a modular form by using different developed toolbox to answer the three main questions, as presented in section 2. The framework is intended to integrate the classic finite element modelling and experimental modal testing used in most industrial application to obtain a successful nonlinear identification. The last phase of the framework seeks to validate the identified nonlinearity through an upgrading approach, where upgrading involves adding valid linear FE models to the nonlinear term to make it complete before updating the coefficients of the parameters. Since there is a need for a comprehensive validation of the nonlinear identification, the upgrading process helps to generate nonlinear models which can be validated by using forced controlled simulation and experimental tests. A schematic of the process is presented in Figure 2.

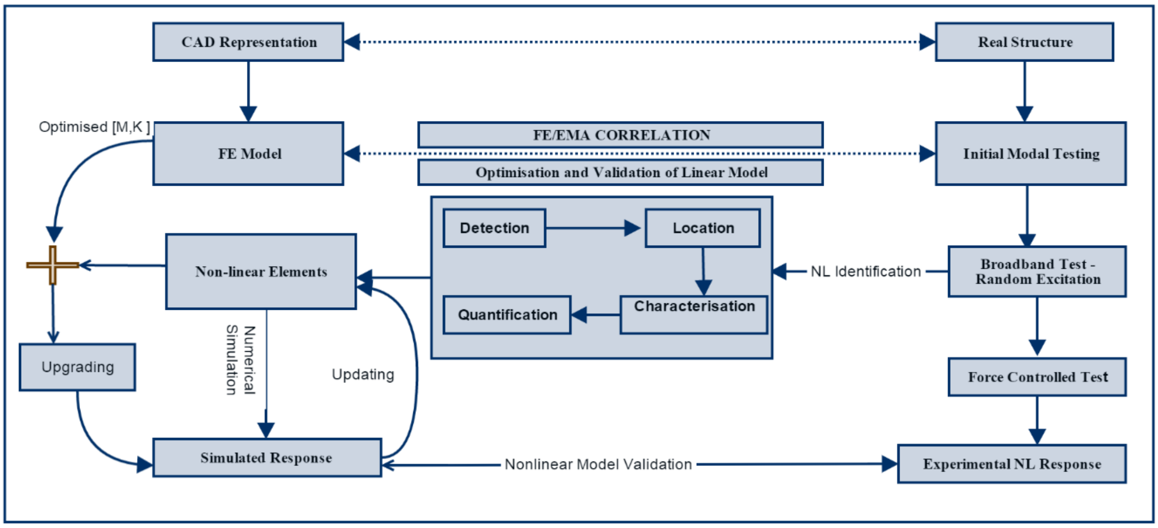

Figure 2: Modular Framework for Model Upgrading

4. The Multiple Beam Structure

The test structure used for this analysis is represented in Figure 3, it is made up of five rectangular steel beams bolted on a large steel frame with M6 bolts. Each beam is connected to each other with the aid of a flexible spring bolted with M4 bolts, the structure is instrumented with an accelerometer close to the tip of each beam and a force gauge attached to beam 3. The structure is designed to display some nonlinear behaviour by tuning the springs with different heights, extra block masses of different weight are added to beam 2- 5 to introduce different dynamics into the system. The double spring connection, between beam 1 and 2, and the flattened spring, between beam 3 and 4, act as sources of nonlinearity.

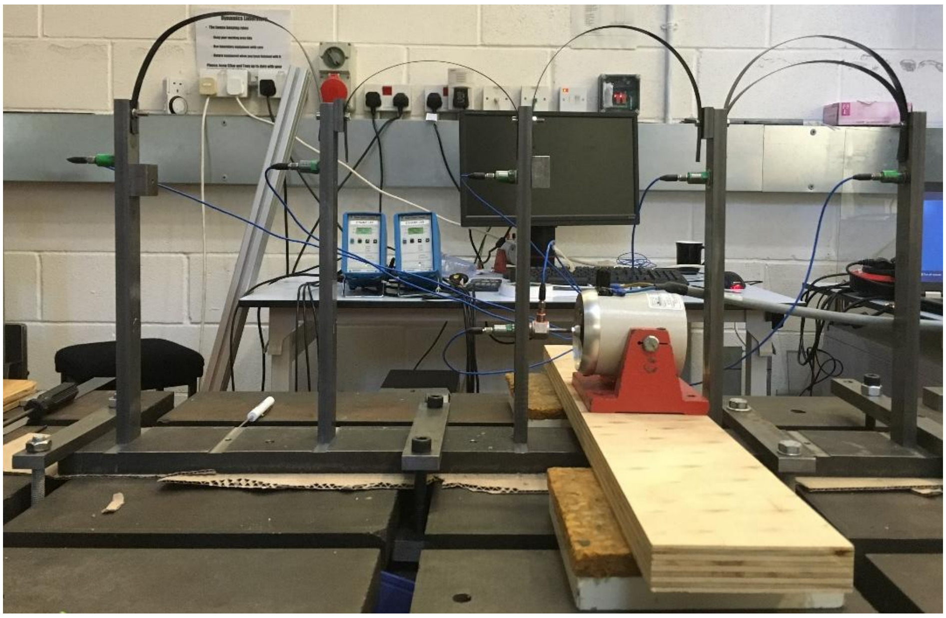

Figure 3: Multiple Beam Test Structure

4.1 *Experimental Testing*

Two different tests were performed on the structure, the first test was a broadband test. The aim of the broadband test is to identify the number of modes present inside a selected frequency range. The second test performed on the structure is a forced control test, the results obtained from this test are used to validate the nonlinear forced simulated model. Low and high level broadband test, using random excitation, were performed on the structure the results of which were able to provide an initial assessment of the structure. Figure 4 and 5 present both the FRF and the coherence for low level of excitation. Figure 6 and 7 present both FRF and coherence for high level of excitation. In this high vibratory state both coherences show clear signs that nonlinearities are being activated.

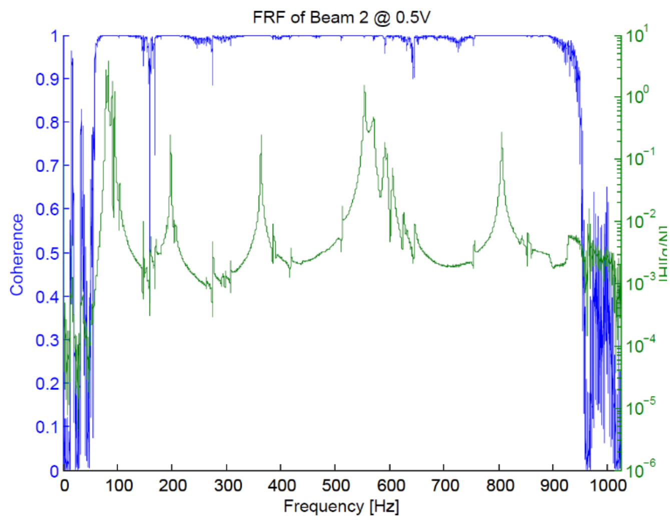

Figure 4: FRF and coherence at low level of excitation beam 2

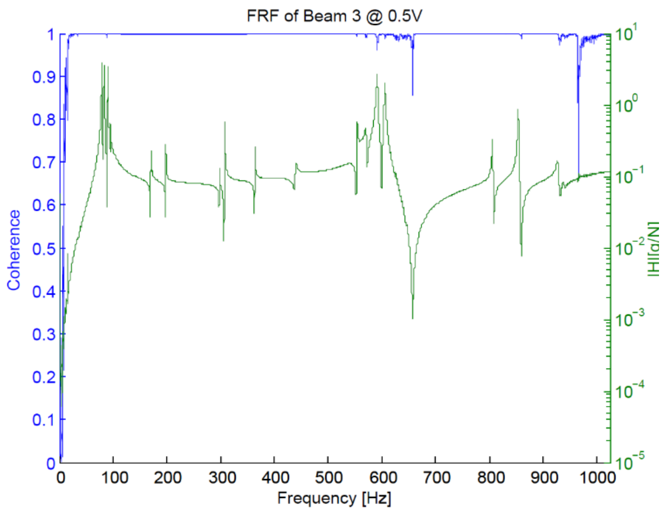

Figure 5: FRF and coherence at low level of excitation beam 3

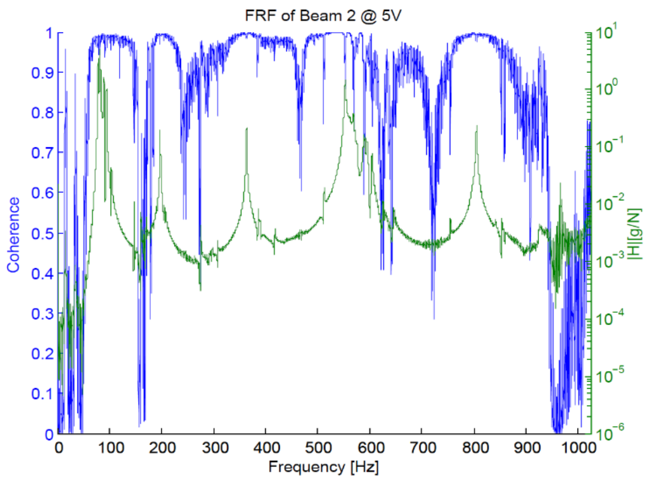

Figure 6: FRF and coherence at high level of excitation beam 2

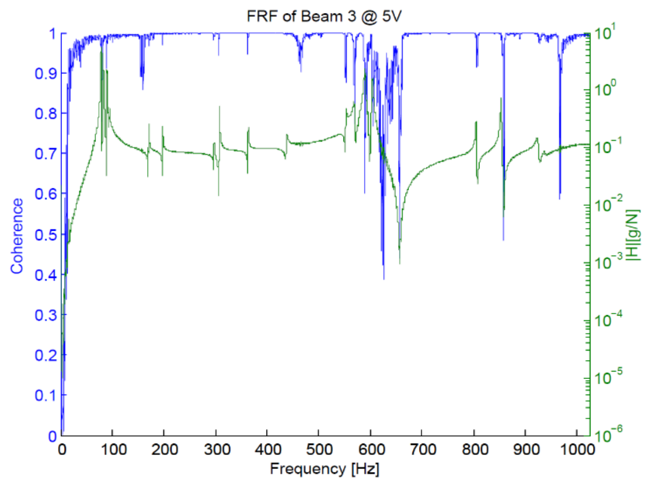

Figure 7: FRF and coherence at high level of excitation beam 3

In total 19 modes were identified within the first 1000[Hz] of the structure. It is also worth mentioning that there other broadband test results for beam 1, 4 and 5 but these results are not included in this paper, in addition as we move further away from the excited beam the energy drops so there the energy that reaches beam 1and 5 are not as high as the other beams.

4.1 *Linear FE Model Validation*

The first phase in the modular frame presented in Figure 2 is to obtain some properties that can represent a valid linear model of the structure, an FE model of the structure was created using the standard ABAQUS software package. The beams were modelled using beam elements in ABAQUS, the flexible springs connecting the beams are modelled using the standard spring and dashpots in ABAQUS toolbox. Initial values of Young's modulus and spring stiffness were assigned to get an initial finite element model before the optimisation process, standard modal analysis was conducted to get the FE frequency values and mode shapes.

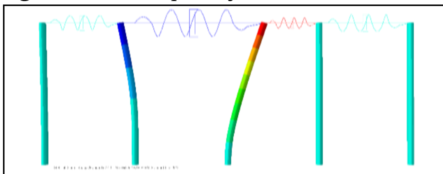

Figure 8: FE Mode 1 at 79.07 [Hz]

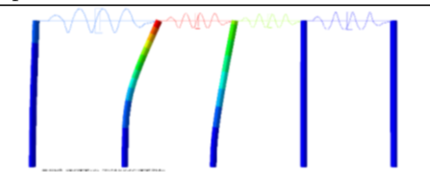

Figure 9: FE Mode 2 at 83.02 [Hz]

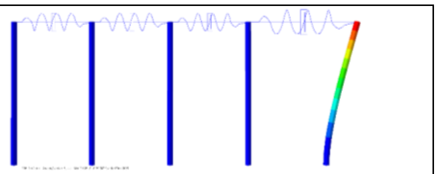

Figure 10: FE Mode 3 at 92.3 [Hz]

Modes	Frequency Value
Mode1	79.07Hz
Mode 2	83.02Hz

Mode 3	92.30Hz

Table 1 Natural frequencies

Figures 8-10 shows the first three FE modes of the structure with their corresponding natural frequencies (see Table 1), although the real springs are modelled as springs and dashpot in the FE model the discrepancy between the FE and experimental natural frequencies are within 3%. The first three modes are mainly considered due to their high resonant peak as shown in Figures 4 to 7.

5. FE and EMA Correlation

To update the mode shapes and natural frequencies of the FE model, standard optimisation software (FEMtools) was used to pair the FE and experimental natural frequencies of the first few modes. The modal assurance criterion (MAC) was used to measure the accuracy of the updated modes with the experimental modes. Table 2 shows the FEA and EMA paired values with their corresponding MAC percentage values.

Pair	FE Freq. (Hz)	EMA Freq.(Hz)	Diff (%)	MAC (%)
1	78.52	78.59	-0.07	64.5
2	83.03	82.95	0.10	52.1
3	92.66	92.55	0.13	74.8
4	103.73	103.74	-0.01	94.8
5	307.98	307.98	0.00	50.5
6	573.61	570.66	0.52	95.9
7	590.90	590.89	0.00	99.5
8	625.82	625.09	0.12	97.0
9	646.03	646.07	-0.01	96.3

Table 2: Updated Natural Frequencies and MAC percentage values

The nature of the joints between the beams (bent steel rulers) makes the system prone to exhibit complex mode shapes which, in turn, make the match of MAC values a much more difficult task to handle, and will lead to bigger MAC errors. These errors, while relatively harmless in a linear environment, will become important when the nonlinear terms are included. As an example, one can think about a nonlinear spring that sits between two degrees of freedom: in a real mode shape there is always a point in which the two degrees of freedom are in their un-deformed position, with the nonlinear spring at rest. In a complex mode shape there is no such point, and the nonlinear spring is always excited in some way. An error on the MAC values means that the mode shapes of the model are not matching the ones of the system, thus the FRF residuals will be off. One can likely find a nonlinear coefficient that accounts for these discrepancies, but in this case its value will not account solely for the nonlinear stiffness of the real joint.

6. Nonlinear Identification

Referring back to the modular framework in Figure 2, the second phase involves the identification of nonlinearities present in the system using the results obtained from the broadband test.

6.1 *Detection*

The first step in the identification process is the detection of nonlinearity, several methods and toolbox have been developed for the detection of nonlinearity but the homogeneity method is by far the most instinctive method using experimental measurement. The method operates by directly comparing 2 FRFs and also relies on the fact that the FRFs of a linear system is independent of its input amplitude. However for a nonlinear system the FRF is dependent on its input force, in this case the results from the stepped-sine and broadband test have shown some evidence of nonlinearity in the structure. Figure 11 and 12 shows FRF of beam 3 at mode 1 and its corresponding cross correlation shift, the homogeneity method must be applied to a mode at a time to return a reliable result hence mode 1 has been chosen for this example.

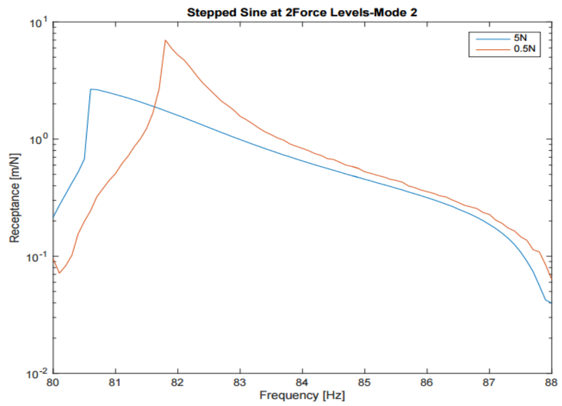

Figure 11: Stepped sine FRF at 2 force levels for mode 2-beam 3

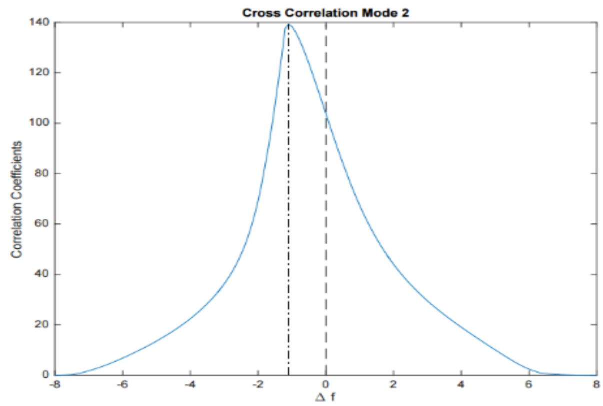

Figure 12: Cross Correlation plot for mode 2 at beam 3

While the homogeneity method has some challenges with quantifying the distortion of the FRF and also providing a physical measure of the nonlinearity in the system, the correlation function in Figure 12 indicates a frequency shift of 1.8[Hz] in addition to a nonlinearity index.

6.1 *Location and Characterisation*

The reverse path method is arguable the most applicable method for locating and characterising nonlinearities based on physical modelling approach, it assumes the nonlinearities as a force feedback terms acting on an underlying linear system[13]. The parameter estimation is performed in the frequency domain using the conventional Multiple-Input-Single-Output (MISO) techniques. Since the overall aim of the modular framework is to conduct all the steps using a physical modelling the reverse path was adopted for this stage of the paper, collecting the broadband time histories responses at high level and treating the nonlinearities as feedback terms the location method is to run a sensitivity analysis about the locations that have the most influence on the coherence of the under-lying linear system. The reverse path relies on the multiple coherence function as an indication for the goodness estimation, Figure 13 shows the coherence sensitivity associated with all the potential nonlinear locations.

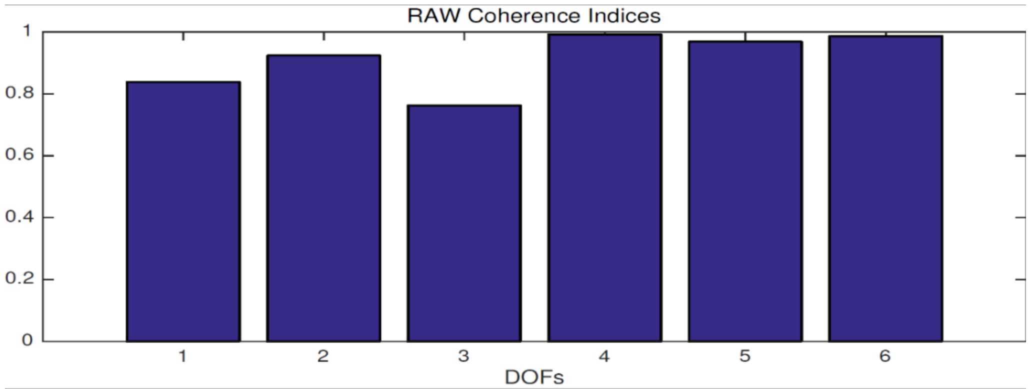

Figure 13: Location plot with all potential nonlinear location

From the chart one can conclude that the most affected DOF is #3, #1 and #2 hence the nonlinearity is located between beams 1 2 and 3. Once an idea of the nonlinear element has been located the reverse path is also used to determine the functional form of nonlinearity in the system, Figure 14 shows the plot for the best iteration of the reverse path where the nonlinear term is characterised to have a forth order function of $F_{NL} = signx^3.|x|$ between DOF #1 and #2. It is important to note that more iteration can be carried out to improve the coherence but at a cost to an increase in the order functional form at the characterisation stage.

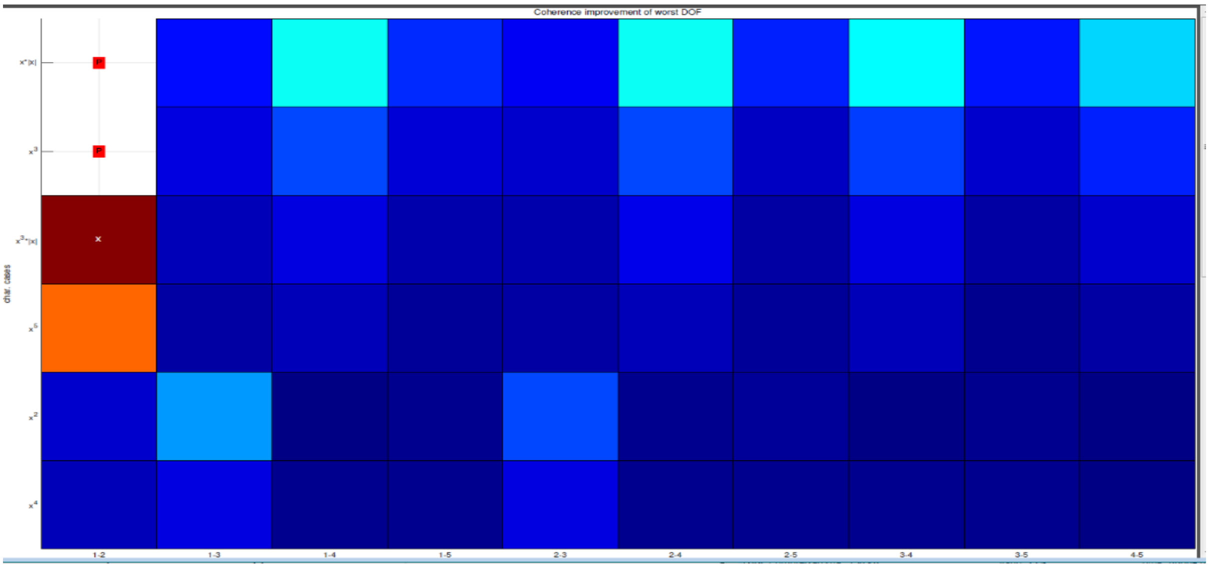

Figure 14: Iteration of characterisation-localisation step using the reverse path

1.1 *Quantification*

After a number of iterations it is possible to plot the final coherence improvement archived during the simulation, figure 15 shows the coherence indices for each degree of freedom, i.e the indices of each beam and the driving point before and after the identification. The coherence function before and after the identification can also be obtained in the bandwidth of interest 80-280[Hz] for DOF #3 which is the worst coherent point as shown in figure 16.

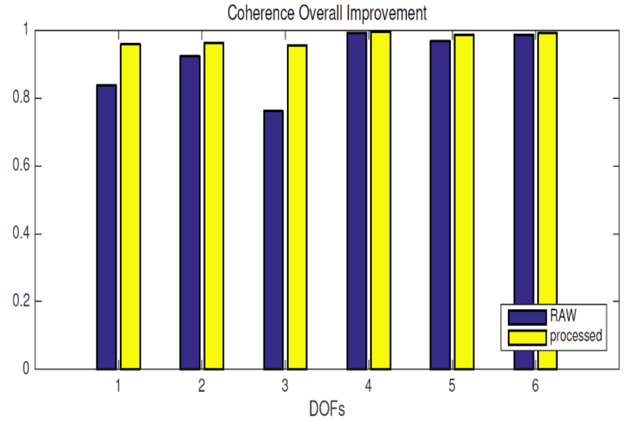

Figure 15: Coherence Improvement after Identification

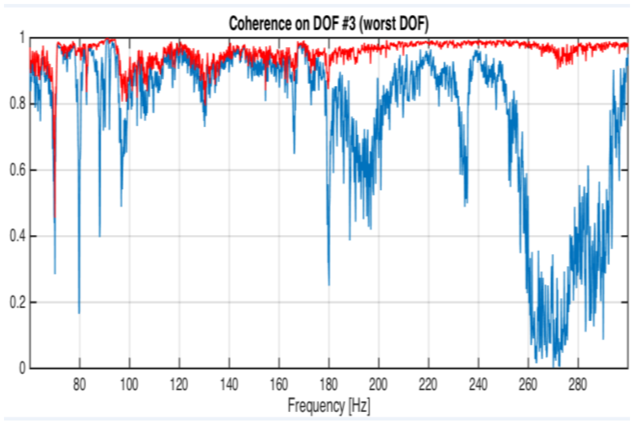

Figure 16: Coherence function at the worst DOF #3

The first initial coefficients of the nonlinearities are also obtainable using the reverse path method as a starting point before going through the process of manual updating. The nonlinear coefficients are expected to be constant and real valued however the reverse path operates in the frequency dependent complex values, therefore the retrieved coefficients are averaged real values. It is worth noting that the retrieved values is obtained based on the information of the underlying linear model and its corresponding averages.

7. Model Upgrading

This section of the modular framework is what brings together all the previous work that has been conducted on the structure, it is often disregarded but it is arguable one of the most important process which helps to complete the structural model in the simulation aspect. The upgrading process entails adding the identified parameters that describe the nonlinear stiffness and damping in the structure to the original validated linear FE model, this is essential to make up a complete model that would describe the nonlinear behaviour within a set range of simulated force values.

The model upgrading was performed on the multiple beam test structure by extracting the optimised mass and stiffness matrices of the linear model and manually added the identified nonlinear terms into the equations of motion used for the numerical simulation. A range of forced controlled simulation was carried out at 0.5N, 3N, 5N and 10N with the full DOF of the FE model, the simulation was conducted in MATLAB using AUTO continuation code. The responses of the model are plotted in units of acceleration and frequency to obtain better curves and continuation path. Figure 19 and 20 shows some example of the response plot of selected nodes from the FE and numerical simulation.

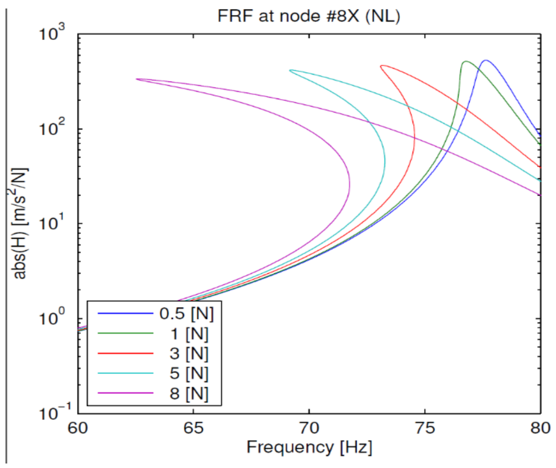

Figure 19: Forced Response Simulation FRF beam 1 @node 8

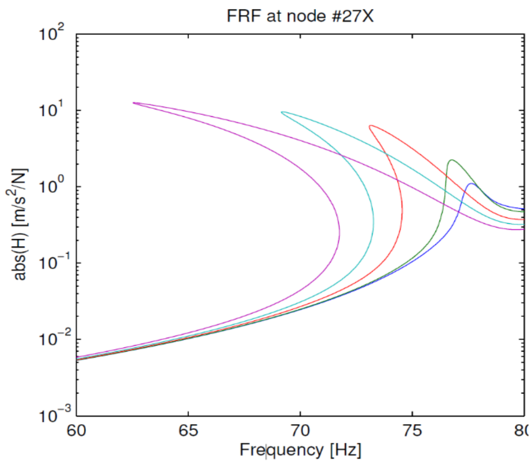

Figure 4: Forced Response Simulation FRF beam 2 @node 27

8. Model Updating

Now that the model has been upgraded, it is still not possible to refer to the model as an exact model but has it now has integrated parameters which can describe the nonlinear behaviour of the real structure. This stage involves matching the initial nonlinear simulated response with the measured response. It is worthing noting that at this stage the functional form of nonlinearity is kept constant and only the coefficients of the nonlinear terms are updated iteratively to match the measured response. To conduct the updating on the structure, forced response simulation was conducted with a frequency range of the first mode using the harmonic balance method at a force level of 8N. A corresponding force control test was conducted on the structure to correlate and update the simulated response. Figure 21 shows the result of the updated response.

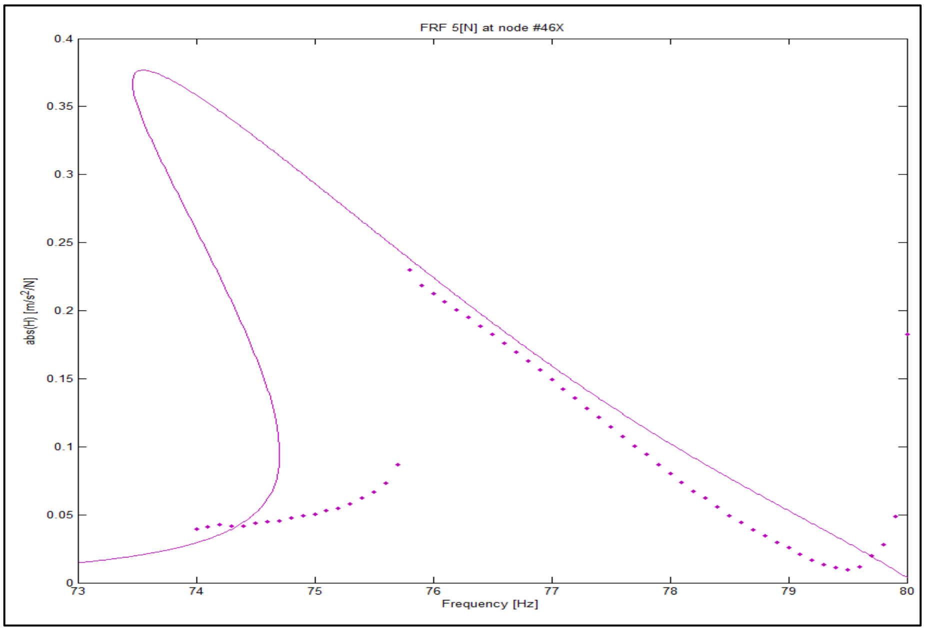

Figure 21: FRF beam 3 measured response and updates simulated response

9. Nonlinear Model Validation

The final step in the modular framework is to validate the identified nonlinear model, this done by generating new forced response simulations using the new coefficients obtained from the updating process. Several forcing levels of [0.5N, 1N, 3N, 5N and 8N] were simulated, the corresponding force control experiment was conducted to obtain measured data. The comparison of these results are shown in figure 22.

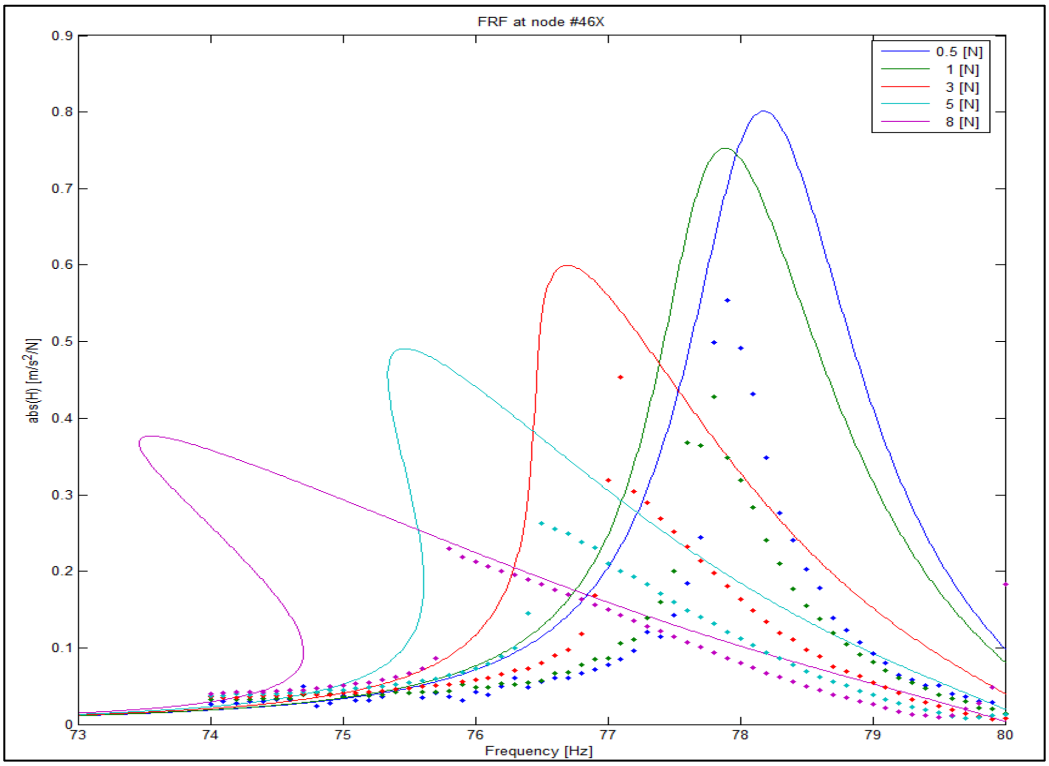

Figure 22: FRF of beam 3 at several forcing levels for model validation

In figure 22 one can observe that there is a frequency shift and the dotted lines which are the measured response are not matching exactly with the continuous line which are the simulated response. This small error can be narrowed back to the underlying linear FE model of the structure, referring back to table 1 which shows the underlying linear frequency and MAC updated values. Although the frequency values have almost zero percent error the MAC values which relates to the mode shapes of the structure are not properly paired during the model updating of linear FE model. Hence this suggests that it is important to have well updated linear model where the mode shapes and frequencies are paired to high degree of accuracy to obtain a well predicted nonlinear model.

10. Conclusion

This paper presents the application of a modular framework for upgrading a valid linear FE model and to accommodate a form of nonlinearity present in the structure. The approach used in the framework were highlighted in three different phases which entails FE/EMA correlation to obtain an underlying linear FE model, nonlinear identification process and model upgrading. The final stage of the application was to validate the identified nonlinear model through simulation and new experimental data. Only a selected number of methods were chosen for the nonlinear identification phase due to the physical modelling approach as adopted for the proposed framework. The error between the simulated and experimental forced responses can be associated to the goodness of the underlying linear FE model and the accuracy of the characterised nonlinear elements. The nonlinear identification and validation phase still remain an area that could benefit from further development most especially using the physical modelling approach.

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